Benha University
Faculty Of Engineering at Shoubra

ECE 122
Electrical Circuits (2)(2017/2018)
Lecture (09)
Transient Analysis (P1)

Prepared By :
Dr. Moataz Elsherbini
motaz.ali@feng.bu.edu.eg

Reference
Chapter 16

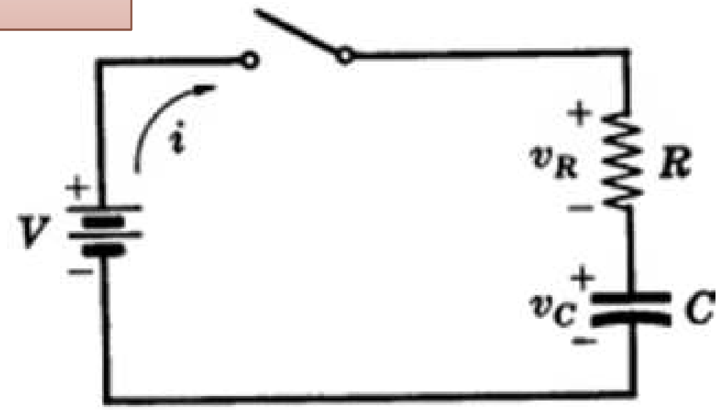
Schaum's Outline Of Theory And Problems Of Electric Circuits

https://archive.org/details/TheoryAndProblemsOfElectricCircuits

1st Order R-C

DC

First-Order RC Transient Step-Response



- Assume the switch S is closed at t = 0
- Apply KVL to the series RC circuit shown:

$$\frac{1}{C}\int i\,dt + Ri = V$$

- Differentiating both sides which gives:

$$\frac{i}{C} + R\frac{di}{dt} = 0 \quad \text{or} \quad \left(D + \frac{1}{RC}\right)i = 0$$

- The solution to this homogeneous equation consists of only the complementary function since the particular solution is zero.
- To find the complementary Solution, solve the auxiliary equation:

$$m + \frac{1}{RC} = 0 \qquad m = \frac{-1}{RC} = \frac{-1}{\tau} \qquad \tau = RC$$

Time constant

The complementary Solution is :

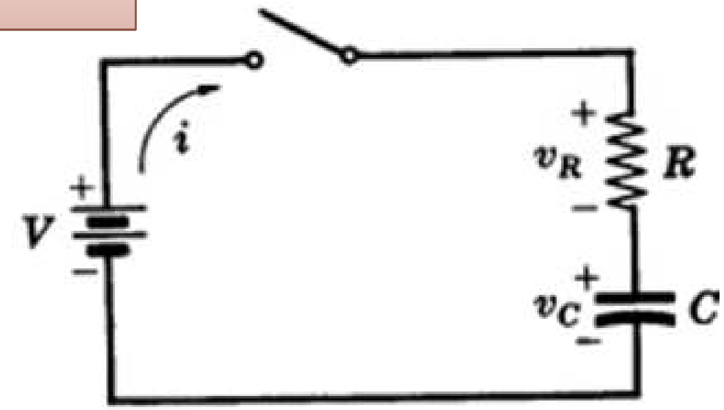
$$i = Ae^{mt} \qquad i = Ae^{\frac{-t}{\tau}}$$

First-Order RC Transient Step-Response

- To determine the constant "A" we note that :

at $t = 0$ is $Ri_0 = V$ or $i_0 = V/R.$

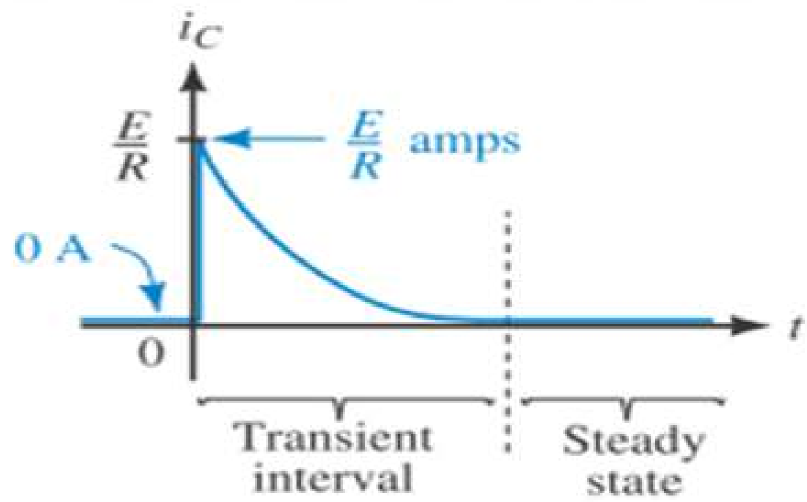
Where Vc (0) = 0



- Now substituting the value of i_o into current equation
- We obtain A = V/R at t = 0.

$$i = \frac{V}{R} e^{-t/RC}$$

has the form of an exponential decay starting from the transient value to the final steady-state value of 0 ampere in 5 time-constants



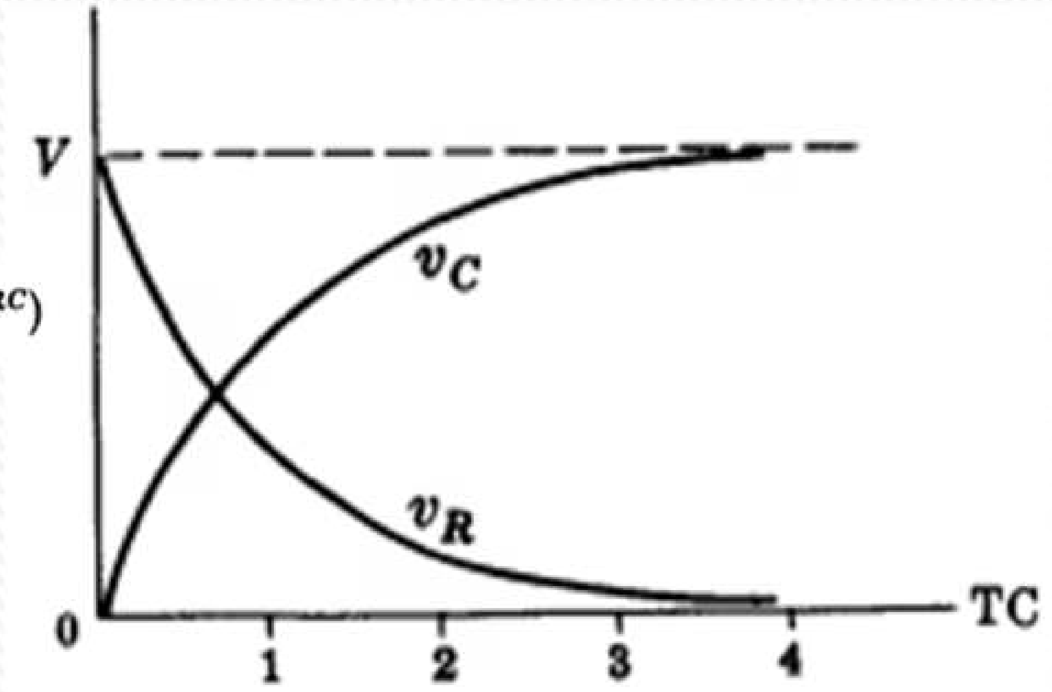
First-Order RC Transient Step-Response

- The voltage across the resistor is:

$$v_R = Ri = Ve^{-t/RC}$$

- The voltage across the capacitor is:

$$v_C = \frac{1}{C}\int i\,dt = V(1 - e^{-t/RC})$$



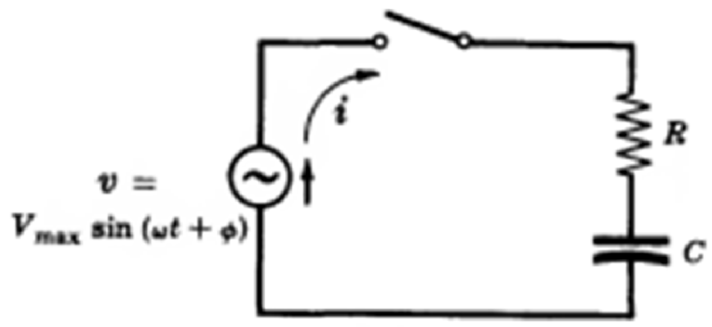
Transient-response is almost finished after 5τ, then steady state

1^{st} Order R-C

AC

Alternating Current Transients

RC Sinusoidal Transient



$$Ri + \frac{1}{C}\int i\,dt = V_{max}\sin(\omega t + \phi)$$

$$\left(D + \frac{1}{RC}\right)i = \frac{\omega V_{max}}{R}\cos(\omega t + \phi)$$

$$i_c = ce^{-t/RC}$$

$$i_p = \frac{V_{max}}{\sqrt{R^2 + (1/\omega C)^2}}\sin(\omega t + \phi + \tan^{-1} 1/\omega CR)$$

$$i = ce^{-t/RC} + \frac{V_{max}}{\sqrt{R^2 + (1/\omega C)^2}}\sin(\omega t + \phi + \tan^{-1} 1/\omega CR)$$

Alternating Current Transients

RC Sinusoidal Transient

To determine the constant c, let $t = 0$ then the initial current $i_0 = \frac{V_{max}}{R} \sin \phi$. Substituting this into (63) and setting $t = 0$, we obtain

$$\frac{V_{max}}{R} \sin \phi = c(1) + \frac{V_{max}}{\sqrt{R^2 + (1/\omega C)^2}} \sin (\phi + \tan^{-1} 1/\omega CR)$$

or

$$c = \frac{V_{max}}{R} \sin \phi - \frac{V_{max}}{\sqrt{R^2 + (1/\omega C)^2}} \sin (\phi + \tan^{-1} 1/\omega CR)$$

Substitution of c from (65) into (63) results in the complete current

$$i = e^{-t/RC} \left[\frac{V_{max}}{R} \sin \phi - \frac{V_{max}}{\sqrt{R^2 + (1/\omega C)^2}} \sin (\phi + \tan^{-1} 1/\omega CR) \right] + \frac{V_{max}}{\sqrt{R^2 + (1/\omega C)^2}} \sin (\omega t + \phi + \tan^{-1} 1/\omega CR)$$

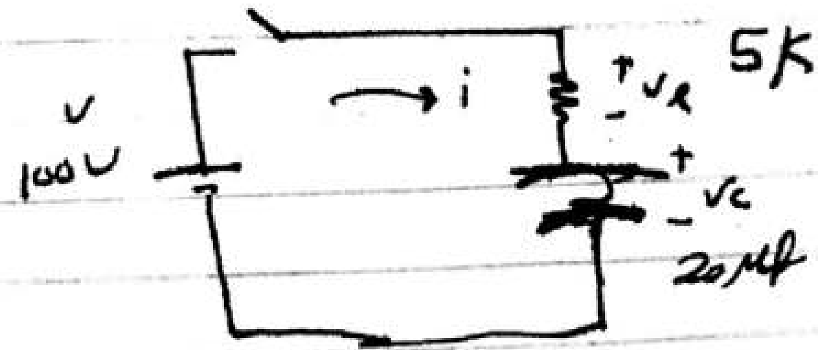
Examples

Example (1)

A series RC circuit with R = 5000 ohms and C = 20 µf has a constant voltage V = 100 v applied at t = 0 and the capacitor has no initial charge. Find the equations of i, V_R and V_{c}.

<u>Sol.</u>

S closed



$$100 = i(5000) + \frac{1}{C}\int i\,dt \rightarrow \boxed{1}$$

$$100 = 5000\,i + \frac{1}{20\times10^{-6}}\int i\,dt$$

$$100 = 5000\,i + 50000\int i\,dt$$

نفاضل الطرفين للتخلص من التكامل

$$0 = 5000\frac{di}{dt} + 50000\,i$$

$$0 = (5000D + 50000)\,i$$

Example (1)

$$(D+10)\, i = 0$$

it has only one solution (P.I)

$$m+10=0$$
$$m=-10$$

$$i = A e^{mt} = A e^{-10t}$$

at $t=0$ → sub in 1 ∴ $100 = 5000\, i$ or $i = 100/50\ldots$

$$i = 0.02A$$

$$\boxed{i = 0.02 e^{-10t}}$$

$$V_R = Ri = 5000 \times 0.02 e^{-10t} = 100 e^{-10t}$$

$$V_C = V - V_R = 100 - 100 e^{-10t}$$

Example (2)

A series RC circuit with R = 100 ohms and C = 25 µf has a sinusoidal voltage source v = 250 sin (500t + Ø) applied at a time when Ø = 0°. Find the current, assuming there is no initial charge on the capacitor.

Sol.

$$100\,i + \frac{1}{C}\int i\,dt = 250 \sin 500t$$

DR

$$(D + 400)\, i = 1250 \cos(500t)$$

بعد التفاضل

Sol

$$i = C e^{-400t} + \frac{V_{max}}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \sin\left(\omega t + \phi + \tan^{-1}\left(\frac{1}{\omega R C}\right)\right)$$

$$i = C e^{-400t} + 1.955 \sin(500t + 38.7^\circ)$$

at $t = 0 \rightarrow i = \frac{250}{100} \sin 0 = 2.5$

نعوض $t = 0$

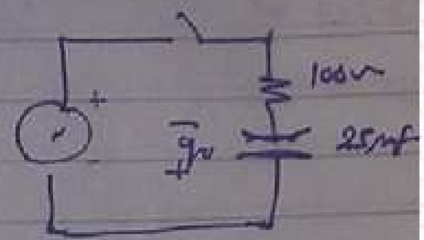
$$\therefore 2.5 = C e^{-400\times 0} + 1.955 \sin(0 + 38.7)$$

$$\therefore C = -1.22$$

$$\therefore i = -1.22\, e^{-400t} + 1.955 \sin(500t + 38.7^\circ)$$

Example (3)

4) in RC circuit shown $v = 250\sin(500t + \phi)$
at $\phi = 45°$ → switch closed, initial charge
$q_0 = 5000 \times 10^{-6}$ Col. on capacitor with shown
polarity, Find I



Sol

∴ من قيم المثال السابق

$$i = c e^{-400t} + 1.955\sin(500t + \phi + 38.7)$$

$$= c e^{-400t} + 1.955\sin(500t + 83.7°)$$

at $t = 0$ $\quad i = \frac{v_{in} + V_c}{100} = \frac{(250\sin 45) + (q_0/c)}{100}$ → 200v

$$i = (250\sin 45° + 200)/100 = 2.77$$

∴ at $t = 0$ → $i = 3.77A$

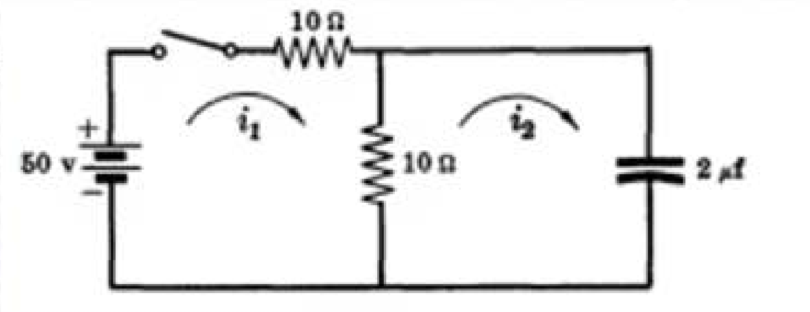
$$3.77 = c e^{0} + 1.955\sin(0 + 83.7°)$$

$$c = 1.83$$

$$\therefore \boxed{i = 1.83 e^{-400t} + 1.955\sin(500t + 83.7°)}$$

Example (4)

In the two-mesh network shown in Fig.4 the switch is closed at t = 0. Find the transient mesh currents i1 and i2 shown in the diagram, and the transient capacitor voltage Vc.



loop 1 $\quad 50 = 20 i_1 - 10 i_2$

$\therefore\ 0 = 20 D i_1 - 10 D i_2$

or $\boxed{2 D i_1 = D i_2}$ ①

بتفاضل الطرفين

Loop 2 $\quad 0 = 10 i_2 - 10 i_1 + \frac{1}{C}\int i_2\, dt$

بتفاضل الطرفين

$0 = 10 D i_2 - 10 D i_1 + \frac{1}{C} i_2$

or $\quad -D i_1 + i_2\left(D + \frac{1}{10C}\right) = 0$ $\quad C \rightarrow 2\times10^{-6}$

$\boxed{-D i_1 + i_2 (D + 5\times10^{4}) = 0}$ → ②

Example (4)

بالتعويض بالمعادلة (1) في (2)

$\therefore -\frac{Di_2}{2} + (D + 5\times10^4)i_2 = 0$

or $(D + 10^5)\, i_2 = 0$

$\therefore$ sol $i_2 = A e^{mt} = A e^{-10^5 t}$

at $t = 0 \longrightarrow$ From eq 2 $\therefore 0 = 10 i_2 - 10 i_1$

$\therefore i_1 = i_2$

From ① $50 = 20 i_1 - 10 i_2 = 10 i_2 = 10 i_1$ والتعويض في ①

$\therefore i_1 = i_2 = 5$

$\therefore$ at $t = 0$ $i_2 = 5 = A e^{0}$ $\therefore A = 5$

or $\boxed{i_2 = 5 e^{-10^5 t}}$

بالتعويض في المعادلة

$50 = 20 i_1 - 10\times5 e^{-10^5 t}$

or $\boxed{i_1 = \frac{5}{2} + \frac{5}{2} e^{-10^5 t}}$

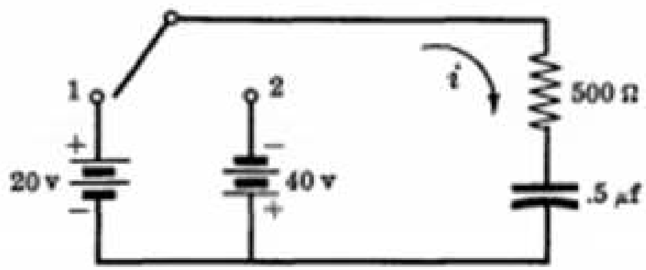
$v_C = \frac{1}{C}\int i_2\, dt$

بتكامل بعد التعويض i_2

$v_C = 25(1 - e^{-10^5 t})$

Example (5)

In the RC circuit of Fig. 3 the switch is closed on position 1 at t=0 and after 1 TC is moved to position 2. Find the complete current transient.



at Position 1

$$20 = 500\, i + \frac{1}{C}\int i\, dt$$

بتفاضل

$$0 = 500 \frac{di}{dt} + \frac{1}{0.5\times 10^{-6}}\, i$$

$$0 = 500\, Di + 2000000\, i$$

$$0 = Di + 4000\, i \qquad i(D + 4000) = 0$$

$$i_1 = A e^{-4000 t_1}$$

Example (5)

at $t=0 \rightarrow \quad i = 20/500 = 0.04 = A$

$\therefore \; i_1 = 0.04\,e^{-4000t_1}$ → يستمر هذا التيار لمدة

$250\,\mu sec = (1RC \quad) = 1TC$

after $1\,TC = 1RC = 250\,\mu sec$

$i = 0.04 \times e^{-4000 \times 250\mu} = 0.147\,A$

0.0147

Now switch moved to 2

$-40 \overset{+\,t_1}{=} 500\,i + \frac{1}{C}\int i\,dt$

$0 = 500\,di/dt + \frac{1}{0.5\times10^{-6}}\,i$

$i_2 = B\,e^{-4000(t-t_1)}$ ← 2

Example (5)

$$\therefore \; v_C = 20\left(1 - e^{-4000t}\right) = 20\left(1 - e^{-4000 \times (RC)}\right) \qquad RC = 250\times10^{-6}$$

1τ = زمن ملاحظة

$$= 20\left(1 - e^{-1}\right) = 12.65 \text{ Volt}$$

$$\therefore \; \boxed{i_2 = B e^{-4000(t-t_1)}}$$

لاحظ أن اتجاه قطبية C

معاكس لاتجاه قطبية البطارية 40V

$$\text{at } t = t_1 \Rightarrow i = \frac{v_{total}}{R}$$

$$i = -\frac{\overset{\text{بطارية}}{40} + \overset{\text{مكثف}}{12.65}}{500} = -0.1053\,A$$

لأن التيار بسبب المكثف ... تيار البطارية عكسه

اتجاه i ← المرسوم

$$\therefore \; \boxed{i_2 = -0.1053\, e^{-4000(t-t_1)}}$$

Example (5)

* (ملاحظة) at $t = t_1 = 250\,\mu sec$

لاحظ إذا رجعنا المفتاح لـ 2 لا زال المكثف محتفظ بالجهد الي عليه

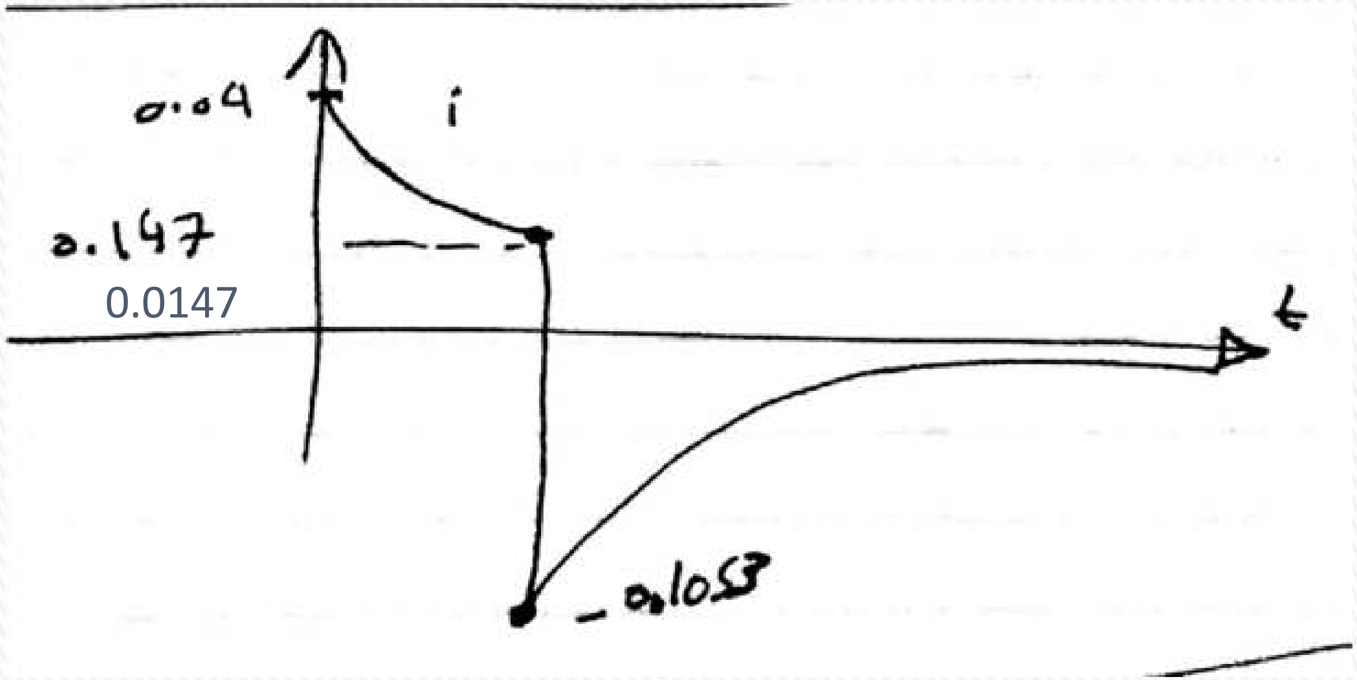
الي هو $\frac{1}{C}\int i\,dt = V_c$ وهو

$$\frac{0.04V}{0.5\times10^{-6}}\,\frac{e^{-4000t}}{-4000} + K = \frac{1}{C}\int i\,dt = V_c$$

$$V_c = -20e^{-4000t} + K$$

at $t=0$ $K=+20$ ($V_c = 0$, $V_R = V$)

$\therefore V_c = 20(1-e^{-4000t}) \rightarrow \tau = RC = 1TC$



Thank You